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More on Permutation Labeling of Graphs

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Abstract

Let $G = (V, E)$ be a graph with order p and size q. An injective function $f : V(G) \to \{1, 2, \ldots, p\}$ is said to be permutation labeling if each edge uv is assigned with label ${}^{f(u)}P_{f(v)} = \frac{(f(u))!}{|f(u)-f(v)|!}$ for $(f(u) > f(v))$ are all distinct. A graph which admits permutation labeling is called permutation graph. In present work we investigate that tree, complete bipartite graph $K_{3,n}$ for $n + 3$ is prime, wheel graph W_n for $n+1$ is prime, dumbbell graph $D_{n,k,2}$ $(n, k \geq 3)$, t^*-ply $P_{t^*}(u, v)$, Petersen graph $P(5, 2)$, crown graph $C_n \odot K_1$ $(n \geq 3)$, one point union $C_n^{(k)}$ of k copies of cycle C_n and middle graph of cycle C_n are combination graph. These results are good applications of combinatorial number theory to graph theory.

Key words: Permutation labeling, t^*-ply $P_{t^*}(u, v)$.

2010 Mathematics Subject Classification : 05C78

1 Introduction

For a graph, if the vertices or edges or both are assigned values subject to certain condition(s) then it is known as graph labeling. Rosa[1] furnished the study of graph labeling in 1967. The last 30 years have witnessed spectacular growth of graph labeling due to its

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wide range of applications in different field such as X-ray crystallography, circuit design, astronomy, coding theory, cryptography and communication networks design. A dynamic survey on graph labeling is regularly updated by Gallian[5] and it is published in *The Electronic Journal of Combinatorics*.
In this paper we consider simple, finite, undirected and connected graph. We refer to Bondy and Murty[4] for the standard terminology and notations related to graph theory and David M. Burton[2] for the terms related to number theory.
Hegde and Shetty[9] defined permutation labeling of graph as follows.

Definition 1.1. An injective function $f : V(G) \to \{1, 2, \ldots, p\}$ is said to be permutation labeling if each edge uv is assigned with label ${}^{f(u)}P_{f(v)} = \frac{(f(u))!}{|f(u)-f(v)|!}$ for $(f(u) > f(v))$ are all distinct. A graph which admits permutation labeling is called permutation graph.

1.1 Some Existing Results on Permutation Graphs

Hegde and Shetty[9] proved that complete graph K_n is permutation graph if and only if $n \leq 5$ and state the **conjecture that every tree is permutation graph.**
Baskar Babujee and Vishnupriya[10] proved that path P_n, cycle C_n, star graph $K_{1,n}$, graph obtained by adding a pendent edge to each edge of star graph and complete binary tree with at least 3 vertices are permutation graph.
Seoud and Salim[6] obtained all permutation graph of order at most 9, they proved that every bipartite graph of order at most 50 is permutation graph. They state the **conjecture that every bipartite graph is permutation graph**.
Shiama[11] derive some permutation graph related to shadow and splitting graph.
Sonchhatra and Ghodasara[8] proved that cycle with one chord, cycle with twin chords, $P_2 + \overline{K_n}$, book graph, tadpole and lotus inside a circle are permutation graphs.

1.2 Some Useful Results of Number Theory

We use the following simple number theory results in this paper.

Lemma 1.2. Let $n, r \in \mathbb{N}$ and $n > r$ then ${}^{n(n+1)\ldots(n+r)}P_1 = {}^{(n+r)} P_{r+1}$

Lemma 1.3. Let $n, m, r, k \in \mathbb{N}$ and $n > m$ and $r > k$ then ${}^nP_r > {}^m P_k$.

Lemma 1.4. Let $n, r, k \in \mathbb{N}$ and $n > k > r$ then ${}^nP_k > {}^k P_r$.

Lemma 1.5. Let $n, r, k \in \mathbb{N}$ and $n > r$ then ${}^nP_r < {}^{n+1} P_r < \ldots < {}^{n+k} P_r$.

Lemma 1.6. Let $n \in \mathbb{N}$ then ${}^nP_{n-1} = n!$ and ${}^nP_1 = n$.

2 Some New Permutation Graphs

The following are the results investigated in this paper.

Theorem 2.1. Every tree is permutation graph.

Proof: Let $u_{0,0}$ be a vertex with maximum degree in a tree T. Choose $u_{0,0}$ as a root vertex. Let l be the height of T.
Let n_1 be the number of vertices at distance one from $u_{0,0}$ and let us denote these vertices by $u_{1,1}, u_{1,2}, \dots u_{1,n_1}$. These vertices are first level vertices.
Let n_2 be the number of vertices at distance two from $u_{0,0}$ which are denoted by $u_{2,1}, u_{2,2}$, $\dots u_{2,n_2}$. These vertices are second level vertices. We give priority as in ascending order.
Repeating this way, let n_l be the number of vertices at distance l from $u_{0,0}$ which are denoted by $u_{l,1}, u_{l,2}, \dots u_{l,n_l}$. These are l^{th} level vertices.
The above process is possible because there is one and only one path between any pair of vertices in a tree.
Here $|V(T)| = \sum_{i=1}^{l}(n_i) + 1 = n$ and $|E(T)| = \sum_{i=1}^{l}(n_i) = n - 1$.
We define bijection $f : V(T) \to \{1, 2, 3, \dots, n\}$ as

$$f(u_{i,j}) = \begin{cases} 1; & i = 0, j = 0. \\ f(u_{i-1,n_{i-1}}) + j; & 1 \leq i \leq l, 1 \leq j \leq \ n_i. \end{cases}$$

The above function f is increasing.
Now for any two edges e_i, e_j in T, there are two cases:
Case 1: If e_i and e_j have common vertex then there are two possibilities:
(1) If e_i and e_j lie on different levels then label of edges are distinct, because of lemma 1.3.
(2) If e_i and e_j lie on same level then label of edges are distinct, because of lemma 1.4.
Case 2: If e_i and e_j have no common vertex then there are two possibilities:
(1) If e_i and e_j lie on different levels then label of edges are distinct, because of lemma 1.2.
(2) If e_i and e_j lie on same level then label of edges are distinct, because of lemma 1.2.
So, from above defined function f, each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ for $f(u) > f(v)$, which are all distinct.
Hence tree T is permutation graph. ∎

Example 2.2. Permutation labeling in tree with 20 vertices is shown in the following figure 1.

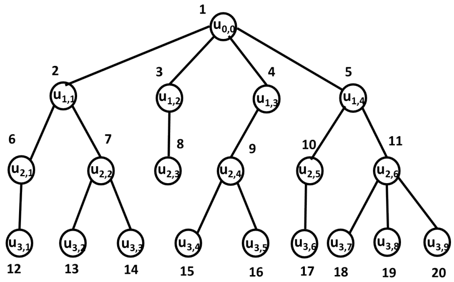

Figure 1

Theorem 2.3. Complete bipartite graph $K_{3,n}$ $(n \geq 1)$ is permutation graph for $n+3$ is prime.

Proof: Let $K_{3,n}$ be the complete bipartite graph with partition of vertex set as $V_1 = \{u_1, u_{n+2}, u_{n+3}\}$ and $V_2 = \{u_2, u_3, \ldots, u_{n+1}\}$.
Here $|V(K_{3,n})| = n+3$ and $|E(K_{3,n})| = 3n$.
We define bijection $f : V(K_{3,n}) \to \{1, 2, 3, \ldots, (n+3)\}$ as
$f(u_i) = i;\ 1 \leq i \leq n+3$.
So from above defined function f, we have three possibilities for label of edges in $K_{3,n}$.
(1) Labels in edge set $\{u_1 u_j,\ j = 2 \ldots n+1\}$ are respectively $2, 3, \ldots n+1$.
(2) Labels in edge set $\{u_{n+2} u_j,\ j = 2 \ldots n+1\}$ are increasing natural number of the form ${}^{(n+2)}P_j$.
(3) Labels in edge set $\{u_{n+3} u_j,\ j = 2 \ldots n+1\}$ are increasing natural number of the form ${}^{(n+3)}P_j$.
Here $f(u_1u_2) < f(u_1u_3) < \ldots f(u_1u_{n+1}) < f(u_{n+2}u_2) < f(u_{n+2}u_3) < \ldots f(u_{n+2}u_{n+1})$ and $(n+3)$ is prime. In above three cases label of edges are internally as well as externally disjoint.
So, from above defined function f, each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ for $f(u) > f(v)$, which are all distinct.
Hence complete bipartite graph $K_{3,n}$ $(n \geq 1)$ is permutation graph when $n+3$ is prime. ■

Example 2.4. Permutation labeling in complete bipartite graph $K_{3,4}$ is shown in the following figure 2.

Definition 2.5 (Bondy and Murty[4]). The wheel graph $W_n (n \geq 3)$ is the graph obtained by joining the graphs C_n and K_1. i.e. $W_n = C_n + K_1$. Here the vertices corresponding

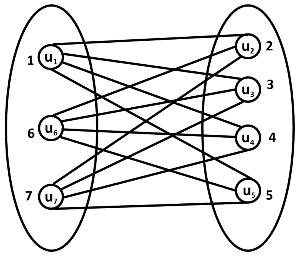

Figure 2

to C_n is called rim vertices and C_n is called rim of W_n while the vertex corresponding to K_1 is called apex vertex.

Theorem 2.6. Wheel graph $W_n(n \geq 3)$ is a permutation graph for $n+1$ is prime.

Proof: Let W_n be a wheel graph with vertex set $\{u_0, u_1, u_2, \ldots, u_n\}$ and an edge set $\{u_0u_i; 1 \leq i \leq n\} \cup \{u_iu_{i+1}; 1 \leq i \leq n-1\} \cup \{u_1u_n\}$, where u_0 be apex and $u_1, u_2, \ldots, u_n$ be rim vertices of wheel graph W_n.
Here $|V(W_n)| = n+1$ (a prime number) and $|E(W_n)| = 2n$.
We define a bijection $f : V(W_n) \to \{1, 2 \ldots, n+1\}$ as follows.

$$f(u_i) = \begin{cases} n+1; & i = 0. \\ 2i-1; & 1 \leq i \leq \frac{n}{2}. \\ n - 2(i - 1 - \frac{n}{2}); & \frac{n+2}{2} \leq i \leq n. \end{cases}$$

Here $f(u_1u_n) < f(u_1u_2) < f(u_nu_{n-1}) < \ldots \; f(u_{(\frac{n+2}{2})}u_{(\frac{n+4}{2})}) < \; f(u_{(\frac{n+2}{2})}u_{(\frac{n}{2})})$.
Since $(n+1)$ is prime, $f(u_0u_i)$ for $1 \leq i \leq n$ is different from other label of edges.
So, from above defined function f, each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ for $f(u) > f(v)$, which are all distinct.
Hence proved. ∎

Example 2.7. Permutation labeling in wheel graph W_{17} is shown in the following figure 3.

Definition 2.8 (Gallian[5])**.** The gear graph $G_n(n \geq 3)$ is a graph obtained from the wheel graph W_n by adding a vertex between every pair of adjacent vertices of the cycle C_n.
$|V(G_n)| = 2n+1$ and $|E(G_n)| = 3n$.

Corollary 2.9. Gear graph G_n $(n \geq 3)$ is a permutation graph for $2n+1$ is prime.

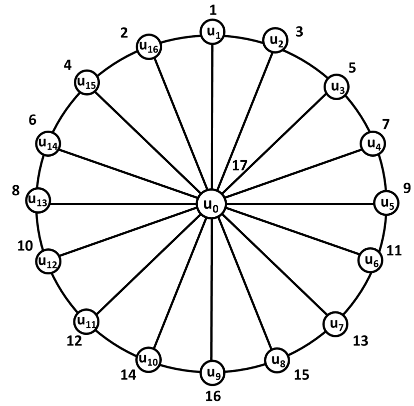

Figure 3

Definition 2.10 (Gallian[5])**.** The shell graph S_n is a graph obtained by taking $n-3$ concurrent chords in a cycle C_n. The vertex at which all the chords are concurrent is called the apex.
A shell graph S_n is also called fan graph F_{n-1}. That is, $S_n = F_{n-1} = P_{n-1} + K_1$.
$|V(S_n)| = n$ and $|E(S_n)| = 2n-3$.

Corollary 2.11. Shell graph S_n $(n \geq 3)$ is permutation graph for n is prime.

Definition 2.12 (Gallian[5])**.** A dumbbell graph $D_{n,k,2}$ $(n, k \geq 3)$ is a graph acquired by joining two cycles C_n and C_k by an edge.
$|V(D_{n,k,2})| = n+k$ and $|E(D_{n,k,2})| = n+k+1$.

Theorem 2.13. Dumbbell graph $D_{n,k,2}$ $(n, k \geq 3)$ is permutation graph.

Proof: Let $u_1, u_2, \ldots u_n$ be the vertices of cycle C_n, where u_i is adjacent to u_{i+1} for $1 \leq i \leq n-1$ and u_n is adjacent to u_1. Let $v_1, v_2, \ldots v_k$ be the vertices of cycle C_k, where v_i is adjacent to v_{i+1} for $1 \leq i \leq k-1$ and v_k is adjacent to v_1.
The dumbbell $D_{n,k,2}$ is a graph obtained by joining C_n and C_k by an edge, say u_1v_1.
We define a bijection $f : V(D_{n,k,2}) \to \{1, 2, 3 \ldots, (n+k)\}$ as per the following cases.
Case 1: n is even.

$$f(u_i) = \begin{cases} n-2i+2; & 1 \leq i \leq \frac{n}{2}. \\ 2i-n-1; & \frac{n+2}{2} < i \leq n. \end{cases}$$

$$f(v_i) = \begin{cases} n+1; & i = 1. \\ n+2i-2; & 1 < i \leq \lfloor \frac{k+2}{2} \rfloor. \\ n+2k-2i+3; & \lfloor \frac{k+2}{2} \rfloor < i \leq k. \end{cases}$$

Case 2: n is odd.

$$f(u_i) = \begin{cases} n; & i = 1. \\ n - 2i + 3; & 1 < i \leq \frac{n+1}{2}. \\ 2i - n - 2; & \frac{n+1}{2} < i \leq n. \end{cases}$$

$$f(v_i) = \begin{cases} n + 1; & i = 1. \\ n + 2i - 2; & 1 < i \leq \lfloor \frac{k+2}{2} \rfloor. \\ n + 2k - 2i + 3; & \lfloor \frac{k+2}{2} \rfloor < i \leq k. \end{cases}$$

From above defined function f, in both the cases, each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ for $(f(u) > f(v))$, which are all distinct because of lemma 1.2, 1.3 and 1.4. Hence Dumbbell graph $D_{n,k,2}$ is permutation graph for all $n, k \geq 3$. ■

Example 2.14. Permutation labeling in Dumbbell graph $D_{15,14,2}$ is shown in the following figure 4.

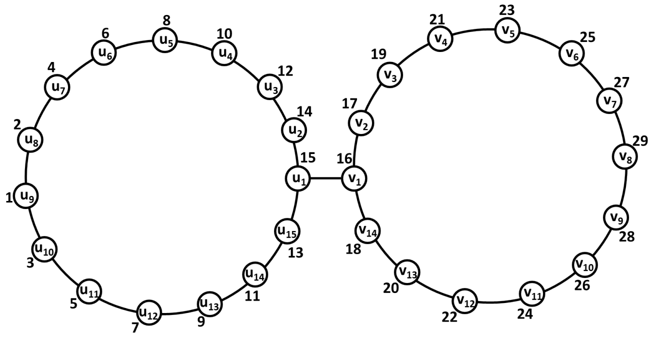

Figure 4

Definition 2.15 (Gallian[5])**.** The corona $H \odot G$ of two graphs H and G is defined as a graph acquired by taking one copy of H (which has h vertices) and h copies of G and attach one copy of G at every vertex of H by an edge.

Theorem 2.16. A crown graph $C_n \odot K_1$ $(n \geq 3)$ is permutation graph.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of cycle C_n, where u_i is adjacent to u_{i+1} $(1 \leq i \leq n-1)$ and u_n is adjacent to u_1. The crown $C_n \odot K_1$ is a graph obtained by joining $u_1, u_2, \ldots, u_n$ to n pendant vertices $v_1, v_2, \ldots, v_n$ respectively.

$|V(C_n \odot K_1)| = 2n$ and $|E(C_n \odot K_1)| = 2n$.

We define a bijection $f : V(C_n \odot K_1) \to \{1, 2, 3 \ldots, 2n\}$ as follows.

$$f(u_i) = \begin{cases} 4i-3; & 1 \le i \le \lfloor \frac{n+1}{2} \rfloor. \\ 4n-4i+3; & \lfloor \frac{n+3}{2} \rfloor \le i \le \ n. \end{cases}$$

$$f(v_i) = \begin{cases} 4i-2; & 1 \le i \le \ \lfloor \frac{n+1}{2} \rfloor. \\ 4n-4i+4; & \lfloor \frac{n+3}{2} \rfloor \le i \le \ n. \end{cases}$$

So, from above defined function f, each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ for $f(u) > f(v)$, which are all distinct because of lemma 1.2, 1.3 and 1.4.
Hence crown graph $C_n \odot K_1$ is permutation graph for $n \ge 3$. ∎

Example 2.17. Permutation labeling in crown graph $C_{15} \odot K_1$ $(n \ge 3)$ is shown in the following figure 5.

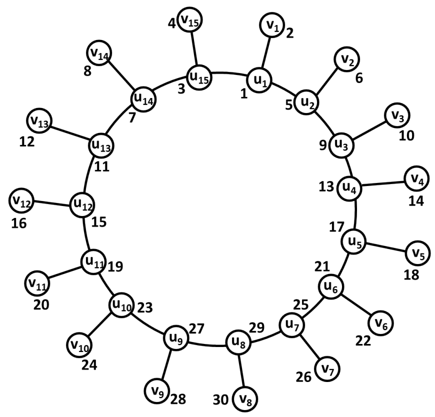

Figure 5

Definition 2.18 (Gallian[5])**.** One point union $C_n^{(k)}$ of k copies of cycle C_n is the graph obtained by taking u as a common vertex such that any two cycles C_n^i and C_n^j $(i \ne j)$ are edge disjoint and do not have any vertex in common except u.

Theorem 2.19. One point union $C_n^{(k)}$ $(k \ge 2, n \ge 3)$ of k copies of cycle C_n is permutation graph.

Proof: Let $u_{j,1}, u_{j,2}, \ldots, u_{j,n}$ be n vertices of j^{th} copy of cycle C_n, $1 \le j \le k$. Note that $u_{1,1} = u_{2,1} = \ldots = u_{k,1}$.
$|V(C_n^{(k)})| = nk - k + 1$ and $|E(C_n^{(k)})| = nk$.
We define a bijection $f : V(C_n^{(k)}) \to \{1, 2, 3, \ldots, (nk - k + 1)\}$ as per the following cases.
Case 1: n is even.

$$f(u_{i,j}) = \begin{cases} 1; & i = j = 1. \\ 2(i + kj - 2k); & 1 \le i \le k, 2 \le j < \frac{n}{2} + 1. \\ i + kn - 2k + 1; & 1 \le i \le k, j = \frac{n}{2} + 1. \\ 2(i + kn - jk) + 1; & 1 \le i \le k, \frac{n}{2} + 1 < i \le \ n. \end{cases}$$

Case 2: n is odd.

$$f(u_{i,j}) = \begin{cases} 1; & i = 1, j = 1. \\ 2(i + kj - 2k); & 1 \le i \le k, 2 \le j < \frac{n+3}{2}. \\ 2i + k(n-3) + 1; & 1 \le i \le k, j = \frac{n+3}{2}. \\ 2(i + kn - jk) + 1; & 1 \le i \le k, \frac{n+3}{2} < i \le \ n. \end{cases}$$

So, from above defined function f, each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ $f(u) > f(v)$, which are all distinct because of lemma 1.2, 1.3 and 1.4.

Hence one point union $C_n^{(k)}$ $(k \ge 2, n \ge 3)$ of k copies of cycle C_n is permutation graph. ∎

Example 2.20. Permutation labeling in one point union $C_8^{(4)}$ of 4 copies of cycle C_8 is shown in the following figure 6.

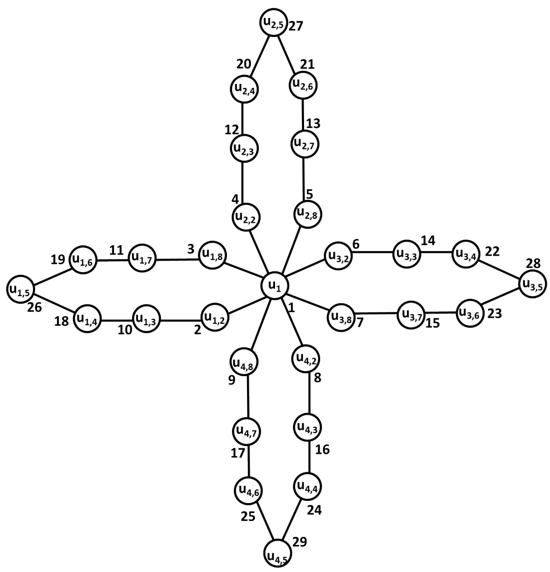

Figure 6

Theorem 2.21. A middle graph $M(C_n)$ of cycle C_n $(n \ge 3)$ is permutation graph.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of cycle C_n, where u_i is adjacent to u_{i+1} for $1 \le i \le n-1$ and u_n is adjacent to u_1. Middle graph $M(C_n)$ of cycle C_n is a graph obtained from C_n by inserting new vertices $v_1, v_2, \ldots, v_n$ corresponding to the edges of cycle C_n, where v_i is adjacent to u_i and u_{i+1} for $1 \leq i \leq n-1$ and v_n is adjacent to u_1 and u_n.

$|V(M(C_n))| = 2n$ and $|E(M(C_n))| = 3n$.

We define a bijection $f : V(M(C_n)) \to \{1, 2, 3 \ldots, 2n\}$ as follows.

$$f(u_i) = \begin{cases} 4i-3; & 1 \leq i \leq \lfloor \frac{n+1}{2} \rfloor. \\ 4n-4i+4; & \lfloor \frac{n+3}{2} \rfloor \leq i \leq n. \end{cases}$$

$$f(v_i) = \begin{cases} 4i-1; & 1 \leq i \leq \lfloor \frac{n+1}{2} \rfloor. \\ 4n-4i+2; & \lfloor \frac{n+3}{2} \rfloor \leq i \leq n. \end{cases}$$

Here $f(u_1v_n) < f(u_1v_1) < f(u_1u_n) < f(u_1u_2) \ldots$.

So, from above defined function f, each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ for $f(u) > f(v)$, which are all distinct because of lemma 1.2, 1.3 and 1.4.

Hence $M(C_n)$ is permutation graph for $n \geq 3$. ∎

Example 2.22. Permutation labeling in middle graph $M(C_{15})$ of cycle C_{15} is shown in the following figure 7.

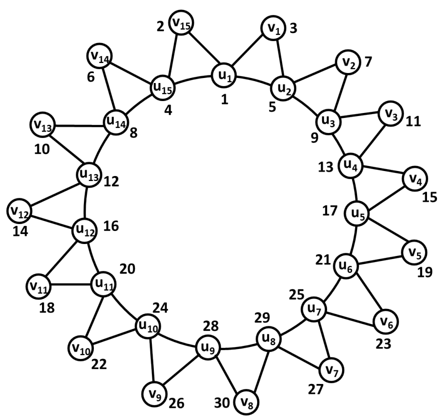

Figure 7

Definition 2.23 (Gallian[5]). A t-ply $P_t(u, v)$ is a graph with t paths, each of length at least two and such that no two paths have a vertex in common except for the end vertices u and v.

A t^*-ply $P_{t^*}(u, v)$ is a special case of t-ply $P_t(u, v)$ graph with every t path have a same length.

Theorem 2.24. A t^*-ply $P_{t^*}(u,v)$ is a permutation graph.

Proof: Let $u_{i,1}, u_{i,2}, \ldots, u_{i,n}$ be the vertices of i^{th} copy of path P_n, $1 \leq i \leq t$.
In t^*-ply graph we identify the vertices $u_{1,1}, u_{2,1}, \ldots, u_{t,1}$ into a single vertex say u_1 and identify the vertices $u_{1,n}, u_{2,n}, \ldots, u_{t,n}$ into a single vertex say u_n.
$|V(P_{t^*}(u,v))| = nt - 2t + 2$ and $|E(P_{t^*}(u,v))| = nt - t$.
We define a bijection $f : V(P_{t^*}(u,v)) \to \{1, 2 \ldots, nt - 2t + 2\}$ as follows.
$f(u_1) = 1$,
$f(u_{i,j}) = t(j-2) + i + 1; \quad 1 \leq i \leq t \ \ , 2 \leq j \leq n-1$.
$f(u_n) = nt - 2t + 2$.
Here $f(u_1 u_{1,2}) \ < f(u_1 u_{2,2}) \ \ldots < f(u_1 u_{t,2}) < f(u_{1,2} u_{1,3}) < f(u_{2,2} u_{2,3}) < \ldots < f(u_{t,n-2}$ $u_{t,n-1}) < f(u_{1,n-1} u_n) \ < f(u_{2,n-1} u_n) \ \ldots < f(u_{t,n-1} u_n)$.
Hence each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ for $f(u) > f(v)$, which are all distinct because of lemma 1.2, 1.3 and 1.4.
Hence t^*-ply $P_{t^*}(u,v)$ is permutation graph. ∎

Example 2.25. Permutation labeling in 5^*-ply $P_{5^*}(u,v)$ is shown in the following figure 8.

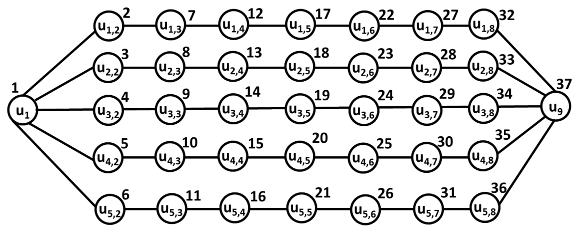

Figure 8

Definition 2.26 (Gallian[5]). The generalized Petersen graph, denoted by $P(n,k)(n \geq 5$, $1 \leq k \leq n)$ is the graph with vertex set $\{a_0, a_1, \ldots a_{n-1}, b_0, b_1, \ldots b_{n-1}\}$ and edge set $\{a_i a_{i+1} \setminus i = 0, 1, \ldots n-1\} \cup \{a_i b_i \setminus i = 0, 1, \ldots n-1\} \cup \{b_i b_{i+k} \ \setminus i = 0, 1, \ldots n-1\}$, where all subscripts are taken over modulo n. The standard Petersen graph is $P(5,2)$.

Theorem 2.27. Petersen graph $P(5,2)$ is permutation graph.

Proof: Let $u_1, u_2, \ldots u_5$ be external and $u_6, u_7, \ldots u_{10}$ be internal vertices of Petersen graph $P(5,2)$ where u_i is adjacent u_{i+5}, $1 \leq i \leq 5$.
Let us define a bijective function $f : V(P(5,2)) \to \{1, 2, \ldots, 10\}$ as follows.

$f(u_i) = i$; $1 \leq i \leq 5$,
$f(u_6) = 10$,
$f(u_i) = i - 1$; $7 \leq i \leq 10$.
Hence each edge uv is identified the label $\frac{(f(u))!}{|f(u)-f(v)|!}$ for $f(u) > f(v)$, which are all distinct. Hence Petersen graph $P(5, 2)$ is permutation graph. ■

Example 2.28. Permutation labeling in Petersen graph $P(5, 2)$ is shown in the following figure 9.

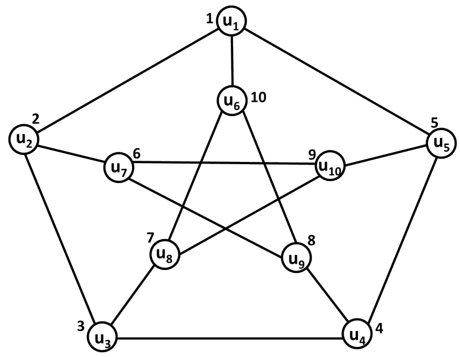

Figure 9

3 Conclusion

Permutation labeling is a bridge connecting combinatorics and graph theory. It is very interesting to study graphs which admit permutation labeling. Here we discovered some permutation graph. To investigate equivalent results for different graph families is an open area of research.

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