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# Some New Sum Divisor Cordial Graphs

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**Abstract**

In this paper we investigate that cycle, cycle with one chord, cycle with twin chord, cycle with triangle, wheel, helm, web, shell, flower, double fan, admit sum divisor cordial labeling.

**Key words:** Divisor cordial labeling, Sum divisor cordial labeling.

**2010 Mathematics Subject Classification** : 05C78.

## 1 Introduction

By a graph, we mean simple, finite, connected and undirected graph. For standard terminology and notations related to graph theory we refer to Gross and Yellen[3], while for number theory we refer to Burton[1].

**Definition 1.1** (Varatharajan et al.[6])**.** Let $G = (V, E)$ be a graph and $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ be a bijection. Consider the induced edge function $f^* : E(G) \to \{0, 1\}$ defined as

$$f^*(uv) = \begin{cases} 1; & \text{if } f(u) \mid f(v) \text{ or } f(v) \mid f(u) \\ 0; & \text{otherwise} \end{cases}$$

The function $f$ is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$.
A graph which admits a divisor cordial labeling is called a divisor cordial graph.

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**Definition 1.2** (A. Lourdusamy and F. Patrick [4])**.** Let $G = (V, E)$ be a graph and $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ be a bijection. Consider the induced function $f^* : E(G) \to \{0, 1\}$ defined as

$$f^*(uv) = \begin{cases} 1; & \text{if } 2 \mid (f(u) + f(v)) \\ 0; & \text{otherwise} \end{cases}$$

The function $f$ is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$.
A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.

## 2 Sum Divisor Cordial Graphs

**Theorem 2.1.** Cycle $C_n$ $(n \in \mathbb{N}, n \geq 3)$ is sum divisor cordial graph, except for $n \equiv 2(mod\ 4)$.

**Proof:** Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$, $|V(C_n)| = n$ and $|E(C_n)| = n$.
We define vertex labeling function $f : V(G) \to \{1, 2, 3, \ldots, n\}$ as follows.
Case 1: For $n \equiv 0, 1, 3(mod\ 4)$.
For $1 \leq i \leq n$:

$$f(v_i) = \begin{cases} i & ; i \equiv 0, 1(mod\ 4) \\ i + 1 & ; i \equiv 2(mod\ 4) \\ i - 1 & ; i \equiv 3(mod\ 4) \end{cases}$$

Case 2: For $n \equiv 2(mod\ 4)$.
then $C_n$ is not sum divisor cordial graph.
In order to satisfy the edge condition for sum divisor cordial graph it is essential to assign label 0 to $\frac{n}{2}$ edges and label 1 to $\frac{n}{2}$ edges out of total $n$ edges.
The edge label will give rise at least $\frac{n}{2} + 1$ edges with label 0 and at most $\frac{n}{2} - 1$ edges with label 1 out of total $n$ edges.
Therefore $|e_f(1) - e_f(0)| = 2$.
Thus the edge condition for sum divisor cordial graph is violated.
Hence $C_n$ is not sum divisor cordial for $n \equiv 2(mod\ 4)$.
In view of above labeling pattern we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 0(mod\ 4)$ | $e_f(0) = e_f(1) = \frac{n}{2}$ |
| $n \equiv 1(mod\ 4)$ | $e_f(0) = \lfloor \frac{n}{2} \rfloor, e_f(1) = \lceil \frac{n}{2} \rceil$ |
| $n \equiv 3(mod\ 4)$ | $e_f(0) = \lceil \frac{n}{2} \rceil, e_f(1) = \lfloor \frac{n}{2} \rfloor$ |

Clearly it satisfies the condition $|e_f(1) - e_f(0)| \leq 1$.
Hence $C_n$ is a sum divisor cordial graph. ∎

**Example 2.2.** Sum divisor cordial labeling of $C_5$ is shown in *Figure 1.*

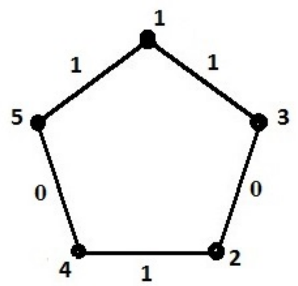


Figure 1:

**Definition 2.3** (J. A. Gallian [2])**.** A **chord** of a cycle $C_n$ $(n \geq 4)$ is an edge joining two non-adjacent vertices.

**Theorem 2.4.** $C_n$ $(n \in \mathbb{N}, n \geq 4)$ with one chord is a sum divisor cordial graph, where chord forms a triangle with two edges of cycle $C_n$.

**Proof:** Let $v_1, v_2, \ldots, v_n$ be the successive vertices of cycle $C_n$ and $e = v_2v_n$ be the chord of $C_n$, where the edges $e_1 = v_1v_2$, $e_2 = v_1v_n$, $e_3 = v_2v_n$ form a triangle.
$|V(G)| = n, \ |E(G)| = n + 1$.
We define vertex labeling $f : V(G) \to \{1, 2, 3, \ldots, n\}$ as follows.
Case 1: For $n \equiv 0, 1, 2(mod\ 4)$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1, 2(mod\ 4) \\ i + 1 & ; i \equiv 3(mod\ 4) \\ i - 1 & ; i \equiv 0(mod\ 4), 1 \leq i \leq n. \end{cases}$$

Case 2: For $n \equiv 3(mod\ 4)$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1, 2(mod\ 4) \\ i + 1 & ; i \equiv 3(mod\ 4) \\ i - 1 & ; i \equiv 0(mod\ 4), 1 \leq i \leq n - 2. \end{cases}$$

$f(v_n) = n-1, f(v_{n-1}) = n.$

In view of above labeling pattern we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 0,2,3(mod\ 4)$ | $e_f(1) = \left\lfloor \frac{n+1}{2} \right\rfloor, e_f(0) = \left\lceil \frac{n+1}{2} \right\rceil$ |
| $n \equiv 1(mod\ 4)$ | $e_f(0) = e_f(1) = \frac{n+1}{2}$ |

Clearly it satisfies the condition $|e_f(1) - e_f(0)| \leq 1$.
Hence, $C_n$ with one chord is a sum divisor cordial graph. ■

**Example 2.5.** Sum divisor cordial labeling of $C_6$ with one chord is shown in *Figure 2*.

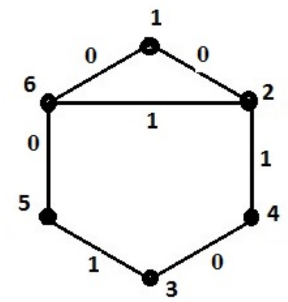


Figure 2:

**Definition 2.6** (J. A. Gallian [2])**.** Two chords of a cycle $C_n$ are said to be **twin chords** if they form a triangle with an edge of $C_n$.
For positive integers $n$ and $p$ with $5 \leq p+2 \leq n$, $C_{n,p}$ denotes cycle $C_n$ with twin chords where chords form cycles $C_p$,$C_3$ and $C_{n+1-p}$ without chords with the edges of $C_n$.

**Theorem 2.7.** Cycle with twin chords $C_{n,3}$ $(n \in \mathbb{N}, n \geq 5)$ is a sum divisor cordial graph, where chords forms two triangles and one cycle $C_{n-2}$.

**Proof:** Let $v_1, v_2, \ldots, v_n$ be the successive vertices of cycle $C_n$ and let $e_1 = v_2v_n$ and $e_2 = v_3v_n$ be the chords of $C_n$. $|V(C_{n,3})| = n$, $|E(C_{n,3})| = n+2$.
We define vertex labeling $f : V(C_{n,3}) \to \{1,2,3,\ldots,n\}$ as follows.
Case 1: $n \equiv 0(mod\ 4)$.
For $1 \leq i \leq n$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1,0(mod\ 4). \\ i+1 & ; i \equiv 2(mod\ 4). \\ i-1 & ; i \equiv 3(mod\ 4). \end{cases}$$

Case 2: $n \equiv 1, 2(mod\ 4)$.

For $1 \leq i \leq n$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1, 2(mod\ 4). \\ i+1 & ; i \equiv 3(mod\ 4). \\ i-1 & ; i \equiv 0(mod\ 4). \end{cases}$$

Case 3: $n \equiv 3(mod\ 4)$

For $1 \leq i \leq n-2$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1, 2(mod\ 4). \\ i+1 & ; i \equiv 3(mod\ 4). \\ i-1 & ; i \equiv 0(mod\ 4). \end{cases}$$

$$f(v_n) = n-1, f(v_{n-1}) = n.$$

In view of above labeling pattern we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 0, 2(mod\ 4)$ | $e_f(0) = \frac{n+2}{2} = e_f(1)$ |
| $n \equiv 1(mod\ 4)$ | $e_f(0) = \left\lfloor \frac{n+2}{2} \right\rfloor, e_f(1) = \left\lceil \frac{n+2}{2} \right\rceil$ |
| $n \equiv 3(mod\ 4)$ | $e_f(1) = \left\lfloor \frac{n+2}{2} \right\rfloor, e_f(0) = \left\lceil \frac{n+2}{2} \right\rceil$ |

Clearly, it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.

Hence, $C_{n,3}$ is a sum divisor cordial graph. ∎

**Example 2.8.** Sum divisor cordial labeling of $C_{7,3}$ is shown in *Figure 3*.

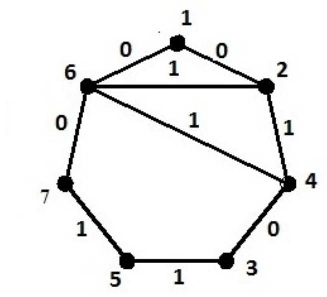


Figure 3:

**Definition 2.9** (J. A. Gallian [2]). A **cycle with triangle** is a cycle with three chords which by themselves form a triangle.
For positive integers $p, q, r$ and $n \geq 6$ with $p + q + r + 3 = n$, $C_n(p, q, r)$ denotes a cycle with triangle whose edges form the edges of cycles $C_{p+2}$, $C_{q+2}$, $C_{r+2}$ without chords.

**Theorem 2.10.** $C_n(1, 1, n-5)$ is a sum divisor cordial graph, except for $n \equiv 3(mod\ 4)$.

**Proof:** Let $v_1, v_2, \ldots, v_n$ be the successive vertices of cycle $C_n$ and $e_1 = v_1v_3$, $e_2 = v_3v_{n-1}$, $e_3 = v_1v_{n-1}$ be the chords of $C_n$.
$|V(C_n(1, 1, n-5))| = n$, $|E(C_n(1, 1, n-5))| = n + 3$.
We define vertex labeling $f : V(G) \to \{1, 2, 3, \ldots, n\}$ as follows.
Case 1: $n \equiv 0, 1(mod\ 4)$.
For $1 \leq i \leq n$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1, 2(mod\ 4) \\ i+1 & ; i \equiv 3(mod\ 4) \\ i-1 & ; i \equiv 0(mod\ 4) \end{cases}$$

Case 2: $n \equiv 2(mod\ 4)$.
For $1 \leq i \leq n-2$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1, 0(mod\ 4) \\ i+1 & ; i \equiv 2(mod\ 4) \\ i-1 & ; i \equiv 3(mod\ 4). \end{cases}$$

$$f(v_n) = n - 1, f(v_{n-1}) = n.$$

Case 3: $n \equiv 3(mod\ 4)$
then cycle with triangle $C_n(1, 1, n-5)$ is not sum divisor cordial graph.
In order to satisfy the edge condition for sum divisor cordial graph it is essential to assign label 0 to $\frac{n+3}{2}$ edges and label 1 to $\frac{n+3}{2}$ edges out of total $n + 3$ edges.
The edge label will give rise at least $\frac{n+3}{2} + 1$ edges with label 0 and at most $\frac{n+3}{2} - 1$ edges with label 1 out of total $n + 3$ edges.
Therefore $|e_f(1) - e_f(0)| = 2$.
Thus the edge condition for sum divisor cordial graph is violated.
Hence cycle with triangle $C_n(1, 1, n-5)$ is not sum divisor cordial for $n \equiv 3(mod\ 4)$.
In view of above labeling pattern we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 1(mod\ 4)$ | $e_f(0) = \frac{n+3}{2} = e_f(1)$ |
| $n \equiv 0(mod\ 4)$ | $e_f(1) = \left\lfloor \frac{n+3}{2} \right\rfloor, e_f(0) = \left\lceil \frac{n+3}{2} \right\rceil$ |
| $n \equiv 2(mod\ 4)$ | $e_f(0) = \left\lfloor \frac{n+3}{2} \right\rfloor, e_f(1) = \left\lceil \frac{n+3}{2} \right\rceil$ |

Clearly it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.
Hence, $C_n(1, 1, n-5)$ is a sum divisor cordial graph. ■

**Example 2.11.** Sum divisor cordial labeling of cycle $C_n(1, 1, 3)$ with triangle is shown in *Figure 4*.

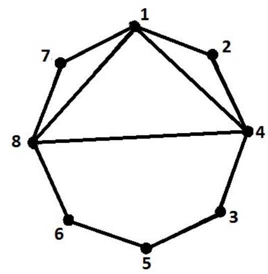


Figure 4:

**Definition 2.12** (J. Gross and J. Yellen [3])**.** The **wheel** graph $W_n(n \in \mathbb{N}, n \geq 3)$ is the join of $C_n$ and $K_1$ i.e. $W_n = C_n + K_1$. Here the edges (vertices) of $C_n$ are called rim edges (rim vertices) and the vertex corresponding to $K_1$ is called apex.

**Theorem 2.13.** *$W_n$ is a sum divisor cordial graph except for $n \equiv 3(mod\ 4)$.*

**Proof:** Let $v_1, v_2, \ldots, v_n$ be the rim vertices and $v_0$ be the apex vertex of $W_n$.
$|V(W_n)| = n + 1$, $|E(W_n)| = 2n$.
We define vertex labeling $f : V(W_n) \to \{1, 2, 3 \ldots, n+1\}$ as follows.
Case 1: $n \equiv 1(mod\ 4)$.

$$f(v_0) = 1.$$

For $1 \leq i \leq n$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 3(mod\ 4) \\ i+1 & ; i \equiv 1, 0(mod\ 4) \\ i+2 & ; i \equiv 2(mod\ 4) \end{cases}$$

Case 2: $n \equiv 0(mod\ 4)$

$f(v_0) = 1.$

For $1 \leq i \leq n$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 3(mod\ 4) \\ i+1 & ; i \equiv 1, 0(mod\ 4) \\ i+2 & ; i \equiv 2(mod\ 4) \end{cases}$$

Case 3: $n \equiv 2(mod\ 4)$

$f(v_0) = 2.$

For $1 \leq i \leq n$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1(mod\ 4) \\ i+1 & ; i \equiv 2, 3(mod\ 4) \\ i+2 & ; i \equiv 0(mod\ 4) \end{cases}$$

Case 4: $n \equiv 3(mod\ 4)$
then $W_n$ is not sum divisor cordial graph.
In order to satisfy the edge condition for sum divisor cordial graph it is essential to assign label 0 to $n$ edges and label 1 to $n$ edges out of total $2n$ edges.
The edge label will give rise at least $n+1$ edges with label 0 and at most $n-1$ edges with label 1 out of total $2n$ edges.
Therefor $|e_f(1) - e_f(0)| = 2$. Thus the edge condition for sum divisor cordial graph is violated.
Hence $W_n$ is not sum divisor cordial, for $n \equiv 3(mod\ 4)$.
In view of above labeling pattern we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 0, 1, 2(mod\ 4)$ | $e_f(0) = n = e_f(1)$ |

Clearly, it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.
Hence, $W_n$ is a sum divisor cordial graph. ∎

**Example 2.14.** Sum divisor cordial labeling of $W_5$ is shown in *Figure 5.*

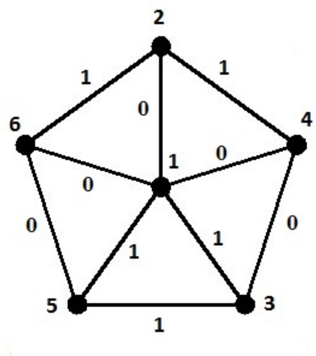


Figure 5:

**Definition 2.15** (J. Gross and J. Yellen [3])**.** The **helm** $H_n(n \in \mathbb{N}, n \geq 3)$ is the graph obtained from a wheel $W_n$ by attaching a pendant edge at each rim vertex of $W_n$.

**Theorem 2.16.** Helm $H_n$ is a sum divisor cordial graph.

**Proof:** Let $v_0$ be the apex vertex, $v_1, v_2, \dots, v_n$ be the vertices of degree 4 and $u_1, u_2, \dots, u_n$ be the pendant vertices of $H_n$.
$|V(H_n)| = 2n+1,\ |E(H_n)| = 3n$.
We define vertex labeling $f : V(H_n) \to \{1, 2, 3, \dots, 2n+1\}$ as follows.

$$f(v_0) = 1.$$

To label the remaining vertices, consider the following cases.
Case 1: $n \equiv 0, 2(mod\ 4)$

$$\begin{aligned} f(v_{2i-1}) &= 4i-1; 1 \leq i \leq \frac{n}{2}. \\ f(v_{2i}) &= 4i-2; 1 \leq i \leq \frac{n}{2}. \\ f(u_i) &= f(v_i)+2; 1 \leq i \leq n. \end{aligned}$$

Case 2: $n \equiv 1, 3(mod\ 4)$

$$\begin{aligned} f(v_{2i-1}) &= 4i-1; 1 \leq i \leq \frac{n-1}{2}. \\ f(v_{2i}) &= 4i-2; 1 \leq i \leq \frac{n-1}{2}. \end{aligned}$$

$$
\begin{aligned}
f(u_i) &= f(v_i)+2; 1 \leq i \leq n-1. \\
f(v_n) &= 2n+1. \\
f(u_n) &= 2n.
\end{aligned}
$$

In view of the above labeling pattern, we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 1,3(mod\ 4)$ | $e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor, e_f(1) = \left\lceil \frac{3n}{2} \right\rceil$ |
| $n \equiv 0,2(mod\ 4)$ | $e_f(0) = \frac{3n}{2} = e_f(1)$ |

Clearly, it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.
Hence $H_n$ is a sum divisor cordial graph. ∎

**Example 2.17.** Sum divisor cordial labeling of the graph $H_6$ is shown in *Figure 6.*

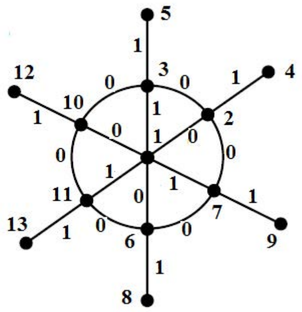


Figure 6:

**Definition 2.18** (J. A. Gallian [2])**. Web** graph $Wb_n(n \in \mathbb{N}, n \geq 3)$ is obtained by joining the pendant vertices of a helm $H_n$ to form a cycle and then adding a pendant edge to each vertex of outer cycle.

**Theorem 2.19.** The Web graph $Wb_n$ is a sum cordial graph.

**Proof:** Let $u_0$ be an apex vertex, $u_1, u_2, \ldots, u_n$ denote the vertices of inner cycle, $v_1, v_2, \ldots, v_n$ denote the vertices of outer cycle and $w_1, w_2, \ldots, w_n$ denote the pendant vertices of $Wb_n$.
$|V(Wb_n)| = 3n+1,\ |E(Wb_n)| = 5n$.
We define vertex labeling $f : V(Wb_n) \to \{1, 2, 3, \ldots, 3n+1\}$ as follows.

$$
\begin{aligned}
f(u_0) &= 1. \\
f(u_i) &= 2i; 1 \leq i \leq n.
\end{aligned}
$$

$$f(v_i) = 2i+1; 1 \le i \le n.$$
$$f(w_i) = (2n+1)+i; 1 \le i \le n.$$

In view of the above labeling pattern, we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 0,2(mod\ 4)$ | $e_f(0) = \frac{5n}{2} = e_f(1)$ |
| $n \equiv 1,3(mod\ 4)$ | $e_f(1) = \left\lfloor \frac{5n}{2} \right\rfloor, e_f(0) = \left\lceil \frac{5n}{2} \right\rceil$ |

Clearly it satisfies the conditions $|e_f(1) - e_f(0)| \le 1$.
Hence, $Wb_n$ is a sum divisor cordial graph. ∎

**Example 2.20.** Sum divisor cordial labeling of $Wb_5$ is shown in *Figure 7.*

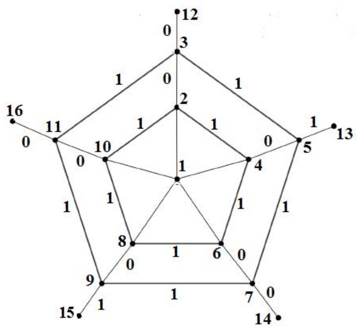

Figure 7:

**Definition 2.21** (J. A. Gallian [2])**.** The **shell** $S_n (n \ge 4, n \in \mathbb{N})$ is the graph obtained by taking $n-3$ concurrent chords in a cycle $C_n$. The vertex at which all the chords are concurrent is called the apex vertex of $S_n$.

**Theorem 2.22.** $S_n$ is a sum divisor cordial graph.

**Proof:** Let $v_1, v_2, \ldots, v_n$ be the vertices of $S_n$, where $v_1$ is the apex vertex.
$|V(G)| = n, |E(G| = 2n-3$.
We define vertex labeling $f : V(S_n) \to \{1,2,3,\ldots,n\}$ as follows.
Case 1: For $n \equiv 1,3(mod\ 4)$, If $1 \le i \le n-1$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1,2(mod\ 4) \\ i+1 & ; i \equiv 3(mod\ 4) \\ i-1 & ; i \equiv 4(mod\ 4) \end{cases}$$

$f(v_n) = n$

Case 2: For $n \equiv 2, 0(mod\ 4)$, If $1 \leq i \leq n$ :

$$f(v_i) = \begin{cases} i & ; i \equiv 1, 2(mod\ 4) \\ i + 1 & ; i \equiv 3(mod\ 4) \\ i - 1 & ; i \equiv 4(mod\ 4) \end{cases}$$

In view of the above labeling pattern, we have

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 0, 2, 3(mod\ 4)$ | $e_f(1) = \left\lfloor \frac{2n-3}{2} \right\rfloor, e_f(0) = \left\lceil \frac{2n-3}{2} \right\rceil$ |
| $n \equiv 1(mod\ 4)$ | $e_f(1) = \left\lceil \frac{2n-3}{2} \right\rceil, e_f(0) = \left\lfloor \frac{2n-3}{2} \right\rfloor$ |

Clearly it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.
Hence, $S_n$ is a sum divisor cordial graph. ∎

**Example 2.23.** Sum divisor cordial labeling of $S_7$ is shown in *Figure 8*.

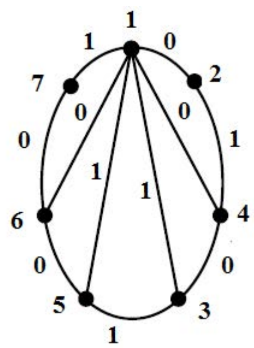


Figure 8:

**Definition 2.24** (J. A. Gallian [2])**.** The **flower** graph $fl_n (n \geq 4, n \in \mathbb{N})$ is obtained from helm $H_n$ by joining each pendant vertex to the central vertex of $H_n$.

**Theorem 2.25.** $fl_n$ is a sum divisor cordial graph.

**Proof:** Let $v_0$ be the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of degree 4 and $u_1, u_2, \ldots, u_n$ be the vertices of degree 2 in $fl_n$.

$|V(fl_n)| = 2n+1, |E(fl_n)| = 4n.$
We define vertex labeling $f : V(fl_n) \rightarrow \{1, 2, 3, \ldots, 2n+1\}$ as follows.

$$\begin{aligned} f(v_0) &= 1. \\ f(v_i) &= 2i; 1 \leq i \leq n. \\ f(u_i) &= 2i+1; 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern, we have

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 0, 1, 2, 3(mod\ 4)$ | $e_f(0) = 2n = e_f(1)$ |

Clearly it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.
Hence, $fl_n$ is a sum divisor cordial graph. ∎

**Example 2.26.** Sum divisor cordial labeling of $fl_4$ is shown in *Figure 9.*

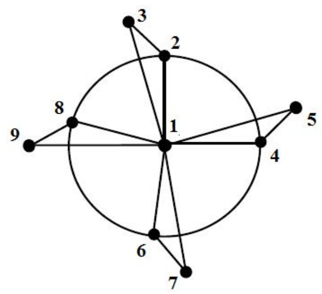


Figure 9:

**Definition 2.27** (J. A. Gallian [2])**.** The **double fan** $Df_n$ $(n \geq 2, n \in \mathbb{N})$ is the graph $P_n + 2K_1$.

**Theorem 2.28.** $DF_n$ is a sum divisor cordial graph.

**Proof:** Let $u, v$ be the apex vertices of degree $n$ and $v_1, v_2, \ldots, v_n$ are vertices on path.
$|V(DF_n)| = n+2, |E(DF_n)| = 3n-1.$
We define vertex labeling $f : V(DF_n) \rightarrow \{1, 2, 3, \ldots, n+2\}$ as follows.

$$\begin{aligned} f(u) &= 1. \\ f(v) &= 2. \end{aligned}$$

Case 1: For $n \equiv 0, 1, 3(mod\ 4)$, If $1 \leq i \leq n$ :

$$f(v_i) = \begin{cases} i+1; & i \equiv 3(mod\ 4) \\ i+2; & i \equiv 1, 0(mod\ 4) \\ i+3; & i \equiv 2(mod\ 4) \end{cases}$$

Case 1: For $n \equiv 2(mod\ 4)$, If $1 \leq i \leq n-1$ :

$$f(v_i) = \begin{cases} i+1; & i \equiv 3(mod\ 4) \\ i+2; & i \equiv 1, 0(mod\ 4) \\ i+3; & i \equiv 2(mod\ 4) \end{cases}$$

$$f(v_n) = n+2.$$

In view of the above labeling pattern, we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 1, 3(mod\ 4)$ | $e_f(0) = \frac{3n-1}{2} = e_f(1)$ |
| $n \equiv 0, 2(mod\ 4)$ | $e_f(1) = \left\lfloor \frac{3n-1}{2} \right\rfloor, e_f(0) = \left\lceil \frac{3n-1}{2} \right\rceil$ |

Clearly it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.
Hence, $DF_n$ is a sum divisor cordial graph. ∎

**Example 2.29.** Sum divisor cordial labeling of $DF_5$ is shown in *Figure 10*.

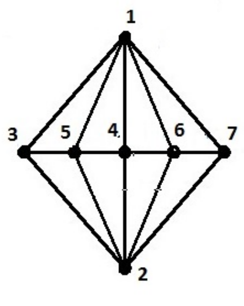


Figure 10:

## 3 Concluding Remarks

In this paper, we derived the sum divisor cordial labeling of cycle $C_n$, $C_n$ with one chord, $C_n$ with twin chords, $C_n$ with triangle, $W_n$, $H_n$, $fl_n$, $Wb_n$, $S_n$, $DF_n$. To investigate analogous results for different graphs and graphs using graph operations is an open area of research. It is also interesting to find the divisor cordial graphs which are sum divisor cordial and vice versa.

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