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# Sum Divisor Cordial Labeling for Vertex Switching of Special Graphs

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**Abstract**

A sum divisor cordial labeling of a graph $G$ with vertex set $V(G)$ is a bijection $f$ from $V(G)$ to $\{1, 2, 3, \ldots, |V(G)|\}$ such that an edge $e = uv$ is assigned the label 1 if $2|[f(u) + f(v)]$ and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph which admits sum divisor cordial labeling is called a sum divisor cordial graph. In this research article we prove that the graphs obtained by switching of a vertex in bistar graph $B_{m,n}$, comb graph $P_n \odot K_1$, apex vertex of helm graph $H_n$, apex vertex of closed helm graph $CH_n$ are sum divisor cordial.

**Key words:** Sum divisor cordial labeling, vertex switching.

**2010 Mathematics Subject Classification** : 05C78

## 1 Introduction

The graphs considered here are finite, connected, undirected and simple. The vertex set and edge set of a graph $G$ are denoted by $V(G)$ and $E(G)$ respectively. For various graph theoretic notations and terminology we follow Gross and Yellen[8] and for standard terminology and notations related to number theory we refer to Burton[5]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

**Remark 1.1.** Throughout this paper $|V(G)|$ and $|E(G)|$ denote the cardinality of vertex set and edge set of graph $G$ respectively.

**Definition 1.2.** If the vertices or edges or both of the graph are assigned valued subject to certain conditions it is known as graph labeling.
Any graph labeling will have the following three common characteristics:

1. A set of numbers from which vertex labels are chosen;

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2. A rule that assigns a value to each edge;

3. A condition that this value has to satisfy.

For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian[7].

**Definition 1.3.** Let $G = (V, E)$ be a simple graph and $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ be a bijection and the induced function $f^* : E(G) \to \{0, 1\}$ be defined as

$$f^*(e = uv) = \begin{cases} 1; & \text{if } f(u) \mid f(v) \text{or} f(v) \mid f(u) \\ 0; & \text{otherwise.} \end{cases}$$

Then $f$ is called divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. Let $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively under $f^*$. A graph with a divisor cordial labeling is called a divisor cordial graph. Varatharajan et al. introduced the concept of divisor cordial labeling of a graph.

They proved that the graphs such as path, cycle, wheel, star and some complete bipartite graphs are divisor cordial graphs. Ghodasara and Adalja [6] derived divisor cordial labeling for vertex switching and duplication of special graphs.

**Definition 1.4.** Let $G = (V, E)$ be a simple graph, $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ be a bijection and the induced function $f^* : E(G) \to \{0, 1\}$ be defined as

$$f^*(e = uv) = \begin{cases} 1; & \text{if } 2 \mid [f(u) + f(v)] \\ 0; & \text{otherwise.} \end{cases}$$

Then $f$ is called sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits sum divisor cordial labeling is called sum divisor cordial graph. A. Lourdusamy, F. Patrick and J. Shiama introduced the concept of sum divisor cordial labeling of graphs.

In[2] they proved shadow graph and splitting graph of $K_{1,n}$, shadow graph, subdivision graph, splitting graph and degree splitting graph of $B_{n,n}$, subdivision graph of ladder, corona of ladder, triangular ladder with $K_1$ and closed helm are sum divisor cordial.
In[3] Adalja and Ghodasara proved that sum divisor cordial labeling of some new graphs. In [4], same author proved that sum divisor cordial labeling for duplication of special graphs.

**Definition 1.5.** The vertex switching $G_v$ of a graph $G$ is the graph obtained by taking a vertex $v$ of $G$, removing all the edges incident to $v$ and adding edges joining $v$ to every other vertex which is not adjacent to $v$ in $G$.

## 2 Main Results

**Definition 2.1.** The Helm graph $H_n$ is the graph obtained from a wheel $W_n$ by attaching a pendant edge to each rim vertex. It contains three types of vertices: an apex of degree $n$, $n$ vertices of degree 4 and $n$ pendant vertices.

**Theorem 2.2.** The graph $G_v$ obtained by switching of apex vertex of helm $H_n$ is a sum divisor cordial graph.

**Proof:** Let $H_n$ be a helm with $v$ as the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of cycle and $u_1, u_2, \ldots, u_n$ be the pendant vertices. Let $G_v$ denotes graph obtained by switching of an apex vertex $v$ of $G = H_n$. Here $|V((H_n)_v)| = 2n+1, |E((H_n)_v)| = 3n$. We define labeling function $f : V((H_n)_v) \rightarrow \{1, 2, \ldots 2n+1\}$ as follows.
For $n \equiv 0, 2(mod 4)$

$$\begin{aligned} f(v) &= 2n+1, \\ f(v_i) &= i; \qquad 1 \leq i \leq n, \\ f(u_i) &= n+1; 1 \leq i \leq n, \end{aligned}$$

For $n \equiv 1, 3(mod 4)$

$$\begin{aligned} f(v) &= 2n, \\ f(v_i) &= \begin{cases} 2i-1 & ; \ i \equiv 1, 3(mod\ 4) \\ 2i-2 & ; \ i \equiv 0, 2(mod\ 4); \quad 1 \leq i \leq n, \end{cases} \\ f(u_i) &= \begin{cases} 2i+1 & ; \ i \equiv 1, 3(mod\ 4) \\ 2i & ; \ i \equiv 0, 2(mod\ 4); \quad 1 \leq i \leq n. \end{cases} \end{aligned}$$

In view of the above labeling pattern we have the following.

| Cases of $n$ | Edge conditions |
|---|---|
| $n \equiv 0, 2(mod\ 4)$ | $e_f(1) = \frac{3n}{2} = e_f(0)$ |
| $n \equiv 1, 3(mod\ 4)$ | $e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor, e_f(1) = \left\lceil \frac{3n}{2} \right\rceil$ |

Thus $|e_f(0) - e_f(1)| \leq 1$.
Hence vertex switching of apex vertex of $H_n$ is a sum divisor cordial graph. ∎

**Example 2.3.** Helm graph $H_6$ and Sum divisor cordial labeling of its switching of apex vertex are shown in Figure 1.

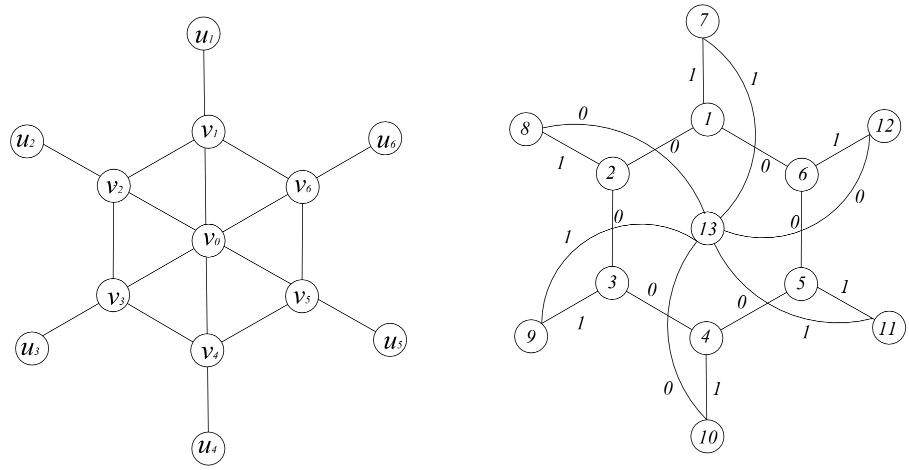


Figure 1

**Definition 2.4.** The closed helm $CH_n$ is the graph obtained from a helm $H_n$ by joining each pendant vertex to form a cycle.

**Theorem 2.5.** The graph $G_v$ obtained by switching of apex vertex of closed helm $CH_n$ is a sum divisor cordial graph.

**Proof:** Let $v$ as the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of inner cycle and $u_1, u_2, \ldots, u_n$ be the vertices of outer cycle $CH_n$. Let $G_v$ denotes graph obtained by switching of an apex vertex $v$ of $G = CH_n$.
Here $|V((CH_n)_v)| = 2n+1, |E((CH_n)_v)| = 4n$.
We define labeling function $f : V((CH_n)_v) \to \{1, 2, \ldots 2n+1\}$ as follows.
For $n \equiv 0, 1, 2, 3(mod4)$

$$\begin{aligned} f(v) &= 1, \\ f(v_i) &= 2i+1; 1 \le i \le n, \\ f(v_i) &= 2i; \quad 1 \le i \le n. \end{aligned}$$

In view of the above labeling pattern we have the following.

$$e_f(1) = 2n = e_f(1).$$

Thus $|e_f(0) - e_f(1)| \leq 1$.
Hence vertex switching of $CH_n$ is a sum divisor cordial graph. ∎

**Example 2.6.** Closed helm graph $CH_6$ and Sum divisor cordial labeling of its switching of apex vertex are shown in Figure 2.

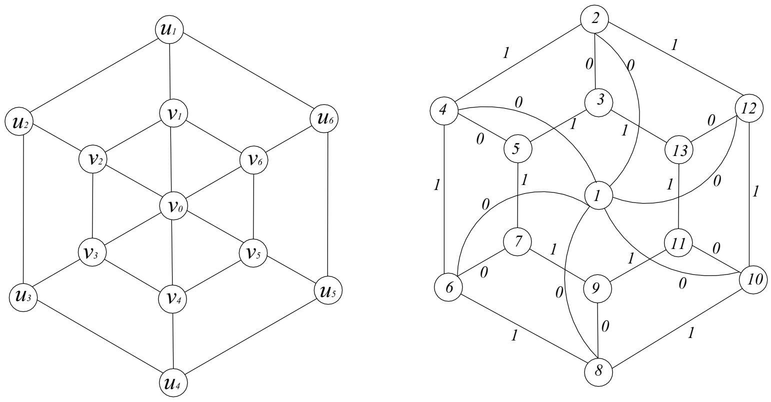


Figure 2

**Definition 2.7.** The bistar graph $B_{m,n}$ is the graph obtained by joining the center(apex) vertices of $K_{1,m}$ and $K_{1,n}$ by an edge.

**Theorem 2.8.** The graph $G_v$ obtained by switching of a vertex in the bistar $B_{m,n}$ is a sum divisor cordial graph.

**Proof:** Let $G = B_{m,n}$ be the bistar with the vertex set $V(G) = \{u_0, v_0, u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$, where $u_0, v_0$ are the apex vertices and $u_i, v_j$ are pendant vertices for all $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. Without loss of generality we are assuming that $m \leq n$ because $B_{m,n}$ and $B_{n,m}$ are isomorphic graphs. Let $G_v$ be the graph obtained by switching of a vertex $v$ in $G$. The proof is divided into following three cases:
**Case 1**: Switching of the pendant vertex.
Without loss of generality we are assuming that the switched pendant vertex is $v_1$. We note that$|V(G_v)| = m + n + 2$ and $|E(G_v)| = 2m + 2n$.
We define vertex labeling $f : V(G_v) \to \{1, 2, \ldots m + n + 2\}$ as follows.

$$\begin{aligned}
f(u_0) &= 2, \quad f(u_1) = 3, \\
f(v_0) &= 4, \quad f(v_1) = 1, \\
f(u_i) &= 3 + i; \qquad 2 \leq i \leq m,
\end{aligned}$$

$$f(v_i) = 3+m+i; \quad 2 \leq i \leq n.$$

In view of the above defined labeling pattern we have $e_f(1) = m+n = e_f(0)$.
Thus $|e_f(0) - e_f(1)| \leq 1$.
**Case 2**: Switching of the vertex of degree $m$.
Here, switched vertex is $u_0$. We note that $|V(G_v)| = m+n+2$ and $|E(G_v)| = 2n$.
We define vertex labeling $f : V((G)_v) \rightarrow \{1, 2, \ldots m+n+2\}$ as follows.

$$\begin{aligned}
f(u_0) &= 2, \\
f(v_0) &= 1, \\
f(u_i) &= 2+n+i; \quad 1 \leq i \leq m, \\
f(v_i) &= 2+i; \qquad 1 \leq i \leq n.
\end{aligned}$$

In view of the above defined labeling pattern we have $e_f(1) = n = e_f(0)$.
Thus $|e_f(0) - e_f(1)| \leq 1$.
**Case 3**: Switching of the vertex of degree $n$.
Here, switched vertex is $v_0$. We note that $|V(G_v)| = m+n+2$ and $|E(G_v)| = 2m$.
We define vertex labeling $f : V((G)_v) \rightarrow \{1, 2, \ldots m+n+2\}$ as follows.

$$\begin{aligned}
f(u_0) &= 2, \\
f(v_0) &= 1, \\
f(u_i) &= 2+n+i; \quad 1 \leq i \leq m, \\
f(v_i) &= 2+i; \qquad 1 \leq i \leq n.
\end{aligned}$$

In view of the above defined labeling pattern we have $e_f(1) = m = e_f(0)$.
Thus $|e_f(0) - e_f(1)| \leq 1$.
Hence, the graph $G_v$ obtained by switching of a vertex in the bistar $B_{m,n}$ is sum divisor cordial.

■

**Example 2.9.** The bistar $G = B_{4,5}$ and the graph $G_v$ obtained by switching of vertex $v_1$ with its sum divisor cordial labeling of are shown in Figure 3.

**Example 2.10.** The bistar $G = B_{4,5}$ and the graph $G_v$ obtained by switching of vertex $u_0$ with its sum divisor cordial labeling of are shown in Figure 4.

**Example 2.11.** The bistar $G = B_{4,5}$ and the graph $G_v$ obtained by switching of vertex $v_0$ with its sum divisor cordial labeling of are shown in Figure 5.

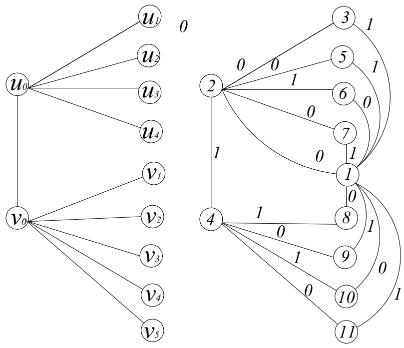

Figure 3

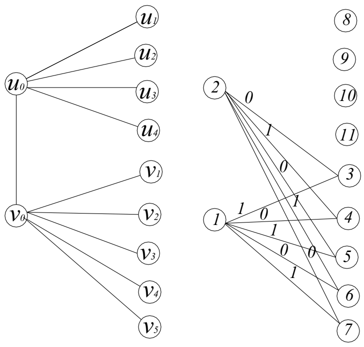

Figure 4

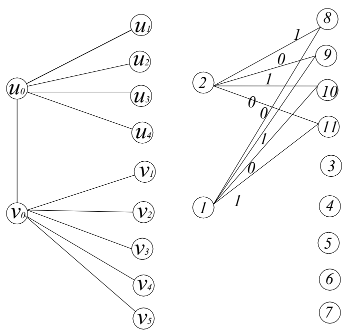

Figure 5

**Definition 2.12.** The comb graph $(P_n \odot K_1)$ is the graph obtained by joining a pendant edge to each vertex of path $P_n$.

**Theorem 2.13.** The graph $G_v$ obtained by switching of any vertex in the comb $P_n \odot K_1$ is a sum divisor cordial graph.

**Proof:** Let $G = P_n \odot K_1$ be the comb graph with the vertex set $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$, where $v_i$ and $u_i$ are pendant and path vertices respectively for all $i = 1, 2, \ldots, n$. Let $G_v$ be the graph obtained by switching of a vertex $v$ in $G$. The proof is divided into following three cases:

**Case 1**: Switching of the pendant vertex.

Without loss of generality we are assuming that the switched pendant vertex is $v_1$. We note that$|V(G_v)| = 2n$ and $|E(G_v)| = 4n - 4$.

We define vertex labeling $f : V(G_v) \to \{1, 2, \ldots 2n\}$ as follows.

$$\begin{aligned} f(u_i) &= 2i - 1; \quad 1 \le i \le n, \\ f(v_i) &= 2i; \qquad 1 \le i \le n. \end{aligned}$$

In view of the above defined labeling pattern we have $e_f(1) = 2n - 2 = e_f(0)$.

Thus $|e_f(0) - e_f(1)| \le 1$.

**Case 2**: Switching of the vertex of degree two.

without loss of generality we are assuming that the switched vertex is $u_1$. We note that $|V(G_v)| = 2n$ and $|E(G_v)| = 4n - 6$.

We define vertex labeling $f : V((G)_v) \to \{1, 2, \ldots 2n\}$ as follows.

$$\begin{aligned} f(u_1) &= 2, \\ f(v_1) &= 1, \\ f(u_i) &= 2i - 1; \quad 2 \le i \le n, \\ f(v_i) &= 2i; \qquad 2 \le i \le n. \end{aligned}$$

In view of the above defined labeling pattern we have $e_f(1) = 2n - 3 = e_f(0)$.

Thus $|e_f(0) - e_f(1)| \le 1$.

**Case 3**: Switching of the vertex of degree three.

without loss of generality we are assuming that the switched vertex is $u_2$. We note that $|V(G_v)| = 2n$ and $|E(G_v)| = 4n - 8$.

We define vertex labeling $f : V((G)_v) \to \{1, 2, \ldots 2n\}$ as follows.

$$\begin{aligned} f(u_1) &= 3, f(u_2) = 2 \\ f(v_1) &= 4, f(v_2) = 1, \\ f(u_i) &= 2i - 1; \quad 3 \le i \le n, \\ f(v_i) &= 2i; \qquad 3 \le i \le n. \end{aligned}$$

In view of the above defined labeling pattern we have $e_f(1) = 2n - 4 = e_f(1)$. Thus $|e_f(0) -$

$e_f(1)| \leq 1$. Hence, the graph $G_v$ obtained by switching of a vertex in the comb $P_n \odot K_1$ is sum divisor cordial. ■

**Example 2.14.** The comb $P_5 \odot K_1$ and the graph $G_v$ obtained by switching of vertex $v_1$ with its sum divisor cordial labeling of are shown in Figure 6.

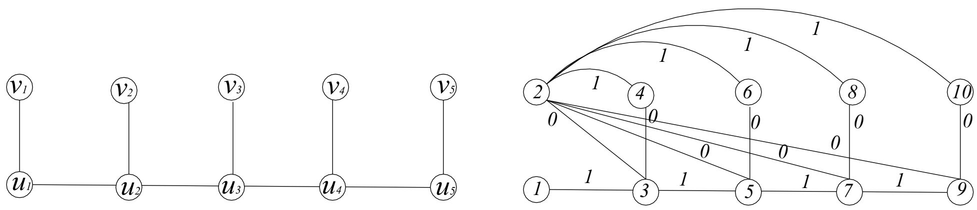


Figure 6

**Example 2.15.** The comb $P_5 \odot K_1$ and the graph $G_v$ obtained by switching of vertex $u_1$ with its sum divisor cordial labeling of are shown in Figure 7.

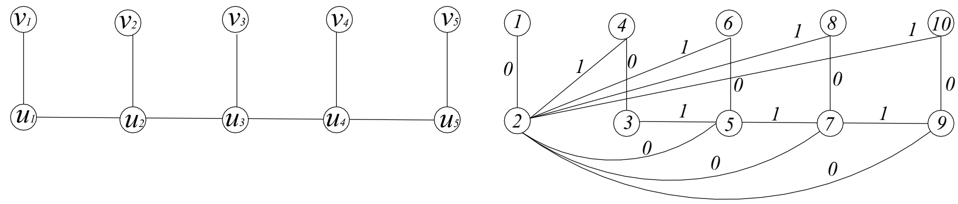


Figure 7

**Example 2.16.** The comb $P_5 \odot K_1$ and the graph $G_v$ obtained by switching of vertex $u_2$ with its sum divisor cordial labeling of are shown in Figure 8.

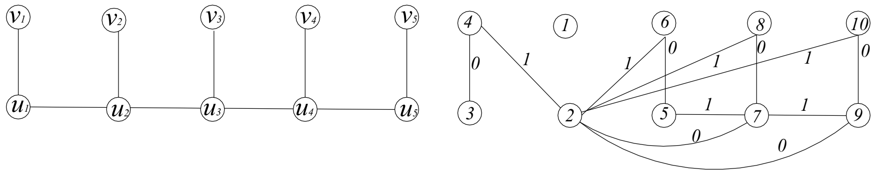


Figure 8

## 3 Concluding Remarks

Here, We have investigated some new results related to the graph operation vertex switching for the sum divisor cordial labeling technique. To explore some new sum divisor cordial graphs in the context of other graph operations is an open area of research.

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